Exercise 32

Explain, using Theorems 4, 5, 7, and 9, why the function is continuous at every number in its domain. State the domain.

$$N(r) = \tan^{-1}(1 + e^{-r^2})$$

Solution

Both 1 and e^{-r^2} are continuous functions at all numbers in their respective domains by Theorem 7, and their sum $1 + e^{-r^2}$ is also a continuous function for all r by Theorem 4. $N(r)$ is a composition of the inverse tangent function and this sum, so by Theorem 9 this is also a continuous function at all numbers in its domain. The inverse tangent of any number can be taken, so the domain of $N(r)$ is

$$(-\infty, \infty).$$